Game Theory

Steven J. Brams
New York University
December 2005
(*International Encyclopedia of the Social Sciences*, 2nd ed., forthcoming)

Game theory is a branch of mathematics used to analyze competitive situations whose outcomes depend not only on one's own choices, and perhaps chance, but also on the choices made by other parties, or "players." Because the outcome of a game is dependent on what *all* players do, each player tries to anticipate the choices of other players in order to determine its own best choice. How these interdependent strategic calculations are made is the subject of the theory.

Game theory was created in practically one stroke with the publication of *Theory of Games and Economic Behavior* in 1944 by mathematician John von Neumann and economist Oskar Morgenstern. This was a monumental intellectual achievement and has given rise to hundreds of books and thousands of articles in a variety of disciplines.

The theory has several major divisions, the following being the most important:

- *2-person versus n-person*: the 2-person theory deals with the optimal strategic choices of two players, whereas the n-person theory ($n > 2$) mostly concerns what coalitions, or subsets of players, will form and be stable, and what constitute reasonable payments to their members.

- *zero-sum versus nonzero-sum*: the payoffs to all players sum to zero (or some other constant) at each outcome in zero-sum games but not in nonzero-sum games, wherein the sums are variable; zero-sum games are games of total conflict, in which what one player gains the others lose, whereas nonzero-sum games permit the players to gain or lose together.

- *cooperative versus noncooperative*: cooperative games are those in which players can make binding and enforceable agreements, whereas noncooperative games may or may not allow for communication among the players but do assume that any agreement reached must be in equilibrium—that is, it is rational for a player not to violate it if other players do not, because the player would be worse off if it did.

Games can be described by several different forms, the three most important being:

(l) *extensive (game tree)*—indicates sequences of choices that players (and possibly chance, according to nature or some random device) can make, with payoffs defined at the end of each sequence of choices;

(2) *normal/strategic (payoff matrix)*—indicates strategies, or complete plans contingent on other players' choices, for each player, with payoffs defined at the intersection of each set of strategies in a matrix;

(3) *characteristic function*—indicates values that all possible coalitions (subsets) of players can ensure for their members, whatever the other players do.

These different game forms, or representations, give less and less detailed information about a game—with the sequences in form l dropped from form 2, and the strategies to implement particular outcomes in form 2 dropped from form 3—to highlight different aspects of a strategic situation.

Common to all areas of game theory is the assumption that players are rational: They have goals, can rank outcomes (or, more stringently, attach utilities, or values, to them), and choose better over worse outcomes. Complications arise from the fact that there is generally no dominant, or unconditionally best, strategy for a player because of the interdependency of player choices. (Games in which there is only one player are sometimes called "games against nature" and are the subject of decision theory.)

A game is sometimes defined as the sum-total of its rules. Common parlor games, like chess or poker, have well-specified rules and are generally zero-sum games, making cooperation with the other player(s) unprofitable. Poker differs from chess in being not only an *n*-person game (though two players can also play it) but also a game of *incomplete information*, because the players do not have full knowledge of each other's hands, which depend in part on chance.

The rules of most real-life games are equivocal; indeed, the "game" may be about the rules to be used (or abrogated). In economics, rules are generally better known and followed than in politics, which is why game theory has become the theoretical foundation of economics, especially microeconomics. But game-theoretic models also play a major role in other subfields of economics, including industrial organization, public economics, and international economics. Even in macroeconomics, in which fiscal and monetary policies are studied, questions about setting interest rates and determining the money supply have a strong strategic component, especially with respect to the timing of such actions. It is little wonder that economics, more than any of the other social sciences, uses game theory at all levels.

Game-theoretic modeling has made major headway in political science, including international relations, in the last generation. While international politics is considered to be quite anarchistic, there is certainly some constancy in the way conflicts develop and may, or may not, be resolved. Arms races, for instance, are almost always nonzero-sum games in which two nations can benefit if they reach some agreement on limiting weapons, but such agreements are often hard to verify or enforce and, consequently, may be unstable.

Since the demise of the superpower conflict around 1990, interest has shifted to whether a new “balance of power”—reminiscent of the political juggling acts of European countries in the nineteenth and early twentieth century—may emerge in different regions or even worldwide. For example, will China, as it become more and more a superpower in Asia, align itself with other major Asian countries, like India and Japan, or will it side more with Western powers to compete against its Asian rivals? Game theory offers tools for studying the stability of new alignments, including those that might develop on political-economy issues.

Consider, for example, the World Trade Organization (WTO), whose durability is now being tested by regional trading agreements that have sprung up among countries in the Americas, Europe, and Asia. The rationality of supporting the WTO, or joining a regional trading bloc, is very much a strategic question that can be illuminated by game theory. Game theory also provides insight into how the domestic politics of a country impinges on its foreign policy, and vice versa, which has led to a renewed interest in the interconnections between these two levels of politics.

Other applications of game theory in political science have been made to strategic voting in committees and elections, the formation and disintegration of parliamentary coalitions, and the distribution of power in weighted voting bodies. On the normative side, electoral reforms have been proposed to lessen the power of certain parties (e.g., the religious parties in Israel), based on game-theoretic analysis. Similarly, the voting weights of members of the European Union Council of Ministers, and its decision rules for taking action (e.g., simple majority or qualified majority), have been studied with an eye to making the body both representative of individual members’ interests and capable of taking collective action.

As game-theoretic models have become more prominent in political science, they have, at the same time, created a good deal of controversy. Some critics charge that they abstract too much from strategic situations, reducing actors to hyper-rational players or bloodless automatons that do not reflect the emotions or the social circumstances of people caught up in conflicts. Moreover, critics contend, game-theoretic models are difficult to test empirically, in part because they depend on counterfactuals that are never observed. That is, they assume that players take into account contingencies that are hard to reconstruct, much less model precisely.

But proponents of game theory counter that the theory brings rigor to the study of strategic choices that no other theory can match. Furthermore, they argue that actors *are*, by and large, rational—they choose better over worse means, even if the goals that they seek to advance are not always apparent.

When information is incomplete, so-called Bayesian calculations can be made that take account of this incompleteness. The different possible goals that players may have can also be analyzed and their consequences assessed.

Because such reconstruction is often difficult to do in real-life settings, laboratory experiments—in which conditions can be better controlled—are more and more conducted. In fact, experiments that test theories of bargaining, voting, and other political-economic processes have become commonplace in economics and political science. Although they are less common in the other social sciences, social psychology has long used experiments to investigate the choices of subjects in games like Prisoners' Dilemma.

This infamous game captures a situation in which two players have dominant strategies of not cooperating, as exemplified by an arms race or a price war. But doing so results in an outcome worse for both than had they cooperated. Because mutual cooperation is not a "Nash equilibrium," however, each player has an incentive to defect from cooperation.

Equally vexing problems confront the players in another well-known game, Chicken. Not only is cooperation unstable, but non-cooperation leads to a disastrous outcome. It turns out that each player should defect if and only if the other player cooperates, but anticipating when an opponent will do so is no mean feat.

Since the invention of game theory more than 60 years ago, its development has been remarkable. Two Nobel prizes in economics were awarded to a total of five game theorists in 1994 and 2005 (including John Nash of "beautiful mind" fame), but many other recipients of this prize have used game theory extensively. In addition, game-theoretic modeling has progressed rapidly in political science—and, to a less extent, in the other social sciences—as well as in a variety of other disciplines, including biology, business, and law.

Bibliography

Aumann, Robert J., and Sergiu Hart (eds.). *Handbook of Game Theory with Economic Applications*. Amsterdam: Elsevier, 1992 (vol. 1), 1994 (vol. 2), 2002 (vol. 3). A comprehensive treatment of game theory and its applications, developed in long review chapters written by leading experts.

Brams, Steven J. *Theory of Moves*. New York: Cambridge University Press, 1994. A dynamic approach to game theory that is applied to a wide variety of conflicts.

Dixit, Avinash, and Susan Skeath. *Games of Strategy*, 2nd ed. New York: W.W. Norton, 2005. An elementary game-theory text that requires only a minimal mathematical background.

Nasar, Sylvia. *A Beautiful Mind*. New York: Simon & Schuster, 1998. A biography of John Nash that also offers an account of the early history of game theory. A fictionalized version of this biography was made into a movie in 2001.

Osborne, Martin J. *An Introduction to Game Theory*. New York: Oxford University Press, 2004. An intermediate text with many applications.

Von Neumann, John, and Oskar Morgenstern. *Theory of Games and Economic Behavior*, 3rd ed. Princeton, NJ: Princeton University Press, 1953. The founding work on game theory by its inventors.